

DIRECT USE OF LINEAR TIME-DOMAIN AERODYNAMICS IN  
AEROSERVOELASTIC ANALYSIS: AERODYNAMIC MODEL

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The work presented here is the first part of a continuing effort to expanding existing capabilities in aeroelasticity by developing the methodology which is necessary to utilize unsteady time-domain aerodynamics directly in aeroservoelastic design and analysis.

The ultimate objective of this study is to define a fully integrated state-space model of an aeroelastic vehicle's aerodynamics, structure and controls which may be used to efficiently determine the vehicle's aeroservoelastic stability.

In this presentation, the current status of developing a state-space model for linear or near-linear time-domain indicial aerodynamic forces is presented.

**MOTIVATION:**

**TO EXPAND EXISTING AEROSERVOELASTIC DESIGN AND ANALYSIS CAPABILITIES TO INCLUDE THE USE OF UNSTEADY TIME-DOMAIN AERODYNAMICS**

**LONG-TERM OBJECTIVE:**

**DEVELOP METHODOLOGY TO UTILIZE LINEAR AND NEAR-LINEAR TIME-DOMAIN AERODYNAMICS IN THE SUPERSONIC AND SUBSONIC REGIMES DIRECTLY IN AEROSERVOELASTIC DESIGN AND ANALYSIS.**

**IMMEDIATE OBJECTIVE:**

**DEVELOP A TIME-DOMAIN STATE-SPACE MODEL OF TIME-DOMAIN AERODYNAMIC INDICIAL FORCES.**

## THE INTEGRATED AEROELASTIC MODEL

To understand the importance of this research, it is necessary to consider that several codes [1,2] have been developed in recent years which compute time-domain unsteady aerodynamics, however, the techniques needed to utilize the aerodynamics in aeroservoelastic design have not been fully developed.

One of the only methods devised to date to evaluate the aeroelastic stability of aerospace vehicles in the time-domain has been a general method capable of handling the nonlinear system [3]. This method is expensive as it involves the computation of the aeroelastic system time response which requires solution of the nonlinear small disturbance aerodynamic equations. Further, a frequency decomposition of the response is necessary to evaluate the stability of component modes. The response must be recomputed at several dynamic pressures until a neutrally stable mode is encountered. Other available methods model the aerodynamics directly in the frequency domain.

For linear and near linear systems in supersonic and subsonic flow, however, the vehicle stability may be evaluated without computing the aeroelastic system forced response or transforming forces to the frequency domain. This is accomplished by representing the time-dependent aerodynamic forces in state-space form coupled with a commonly used state-space representation of the structure. Stability is determined by the eigenvalues of the coupled system matrix.

The focus of this presentation is, again, on the formulation of the aerodynamic portion of the integrated model.

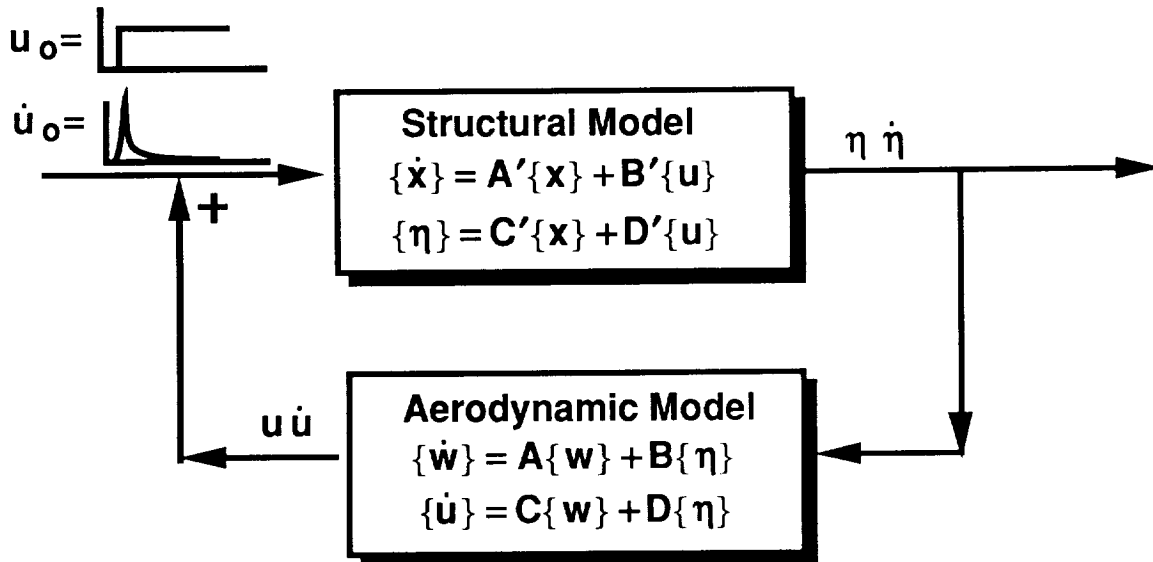


Figure 1. Schematic block diagram indicating integration of the aerodynamic model with the structural model.

## FORMULATION OF AERODYNAMIC MODEL

The aerodynamic model is derived as the Laplace transform of a commonly used frequency domain approximation modified from ref. 4. It is transformed directly into state-space form.

### MODIFIED FREQUENCY DOMAIN APPROXIMATION

$$\bar{Q}(s) = \left( A_0 + A_1 \frac{b}{V} s + \sum_{i=1}^N \frac{B_i s}{s + \beta_i \frac{V}{b}} \right) \eta(s) \bar{q}$$

### STATE-SPACE REPRESENTATION IN TIME DOMAIN

$$\begin{Bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \vdots \\ \dot{w}_N \end{Bmatrix} = \begin{bmatrix} \beta_1 \frac{V}{b} & 0 & \dots & 0 \\ 0 & \beta_2 \frac{V}{b} & & \vdots \\ & & \ddots & \\ 0 & \dots & & \beta_N \frac{V}{b} \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{Bmatrix} + \begin{bmatrix} 0 & B_1 \\ 0 & B_2 \\ \vdots & \vdots \\ 0 & B_N \end{bmatrix} \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix}$$

$$\bar{Q}(t) = \bar{q} [I \ I \ \dots \ I] \begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{Bmatrix} + \bar{q} \begin{bmatrix} A_0 & A_1 \frac{b}{V} \end{bmatrix} \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix}$$

## APPROXIMATION METHOD

The approximation method involves a least squares approximation to the actual aerodynamic force to determine the scalars  $A_0$ ,  $A_1$  and  $B_i$ . The fit is constrained at  $t=0$  to fit exactly and at large times to equal the asymptotic value of the generalized force. As in the frequency-domain rational function type approximations, aerodynamic poles,  $\beta_i$ , are initially specified.

The aerodynamic forces currently being approximated are the rigid-body forces acting on a NACA0064 airfoil and are due to Dowell [5].

### APPROXIMATING FUNCTION

$$\bar{Q}(t) = \left( A_0 \eta(t) + A_1 \frac{b}{V} \dot{\eta}(t) + \sum_{i=1}^N B_i e^{\beta_i \frac{V}{b} t} \right) \bar{q}$$

### CONSTRAINTS ON LEAST SQUARES APPROXIMATION

$$A_0 = Q(t_\infty)$$

$$A_1 = \left( Q(0) - \sum_{i=1}^N B_i - Q(t_\infty) \right) \frac{V}{b}$$

## APPROXIMATION METHOD (CONTI)

A system identification technique frequently used in control system analysis is applied to regenerate the generalized aerodynamic force. Specifically, impulse and step responses of the aerodynamic model are generated using a discrete-time state-transition method. The sum of these responses is the aerodynamic approximation,  $\bar{Q}(t) \equiv Q(t)$ , based on previously determined coefficients and the specified aerodynamic poles.

Due to the discontinuity at  $t=0$  in the impulse input, an assumption is made that at  $t=0+$ , initial conditions are real valued. At  $t=0-$ , initial conditions are zero. This assumption can be shown mathematically.

### STATE TRANSITION EQUATIONS

$$\mathbf{w}(t+1) = \Phi \mathbf{w}(t) + \Gamma \mathbf{u}(t)$$

$$\bar{\mathbf{Q}}(t) = \mathbf{C} \mathbf{w}(t) + \mathbf{D} \mathbf{u}(t)$$

WHERE

$$\Phi(t) = \mathbf{e}^{[\mathbf{A}]T} \quad \text{and} \quad \Gamma = \int_0^T \mathbf{e}^{[\mathbf{A}]\tau} \mathbf{B} d\tau$$

ASSUMING

$$\mathbf{w}(0^-) = \mathbf{0} \qquad \mathbf{w}(0^+) = \mathbf{B} \mathbf{u}(0)$$

## APPROXIMATION METHOD (CONTI)

Improvements to the aerodynamic approximation are made by updating the aerodynamic poles,  $\beta_i$ , followed by another least squares approximation to recompute the coefficients. To update the poles, the method used by Peterson and Crawley [7] to approximate unsteady aerodynamics in the frequency domain is implemented in the time domain. A norm square-error cost function is defined. In this case, the square of the difference between the actual aerodynamic force and the approximation is used. The incremental change in aerodynamic poles is solved for by inverting the Hessian,  $\partial^2 J / \partial \beta_i \partial \beta_k$ , in a single term Taylor series expansion of  $\partial J / \partial \beta$ . The incremental change in  $\beta_i$  is multiplied by a scale factor,  $\alpha$ , and added to the current aerodynamic poles. The scale factor,  $\alpha$ , is computed using quadratic interpolation [8] to insure that the cost is approaching a local extrema.

The new aerodynamic poles are limited. If a given pole is greater than -0.01, it is set equal to that value until the next parameter update. To prevent a pole from going to  $-\infty$  and ill-conditioning the system matrix later on, the pole is limited to a value which would produce no more than a 99.5% decrease in magnitude of the exponential over a given time step.

The two step procedure of computing system coefficients and updating aerodynamic states is repeated until the cost function has been minimized.

## SQUARE ERROR COST FUNCTION

$$J(\beta) = [\mathbf{Q}(t) - \bar{\mathbf{Q}}(t, \beta)]^T [\mathbf{Q}(t) - \bar{\mathbf{Q}}(t, \beta)]$$

## NEWTON RAPHSON STEP

$$\{\delta \beta_i\} = [\mathbf{H}]^{-1} \left\{ \frac{\partial J}{\partial \beta_i} \right\}$$

WHERE,

$$\mathbf{H} = \frac{\partial^2 J}{\partial \beta_i \partial \beta_k}$$

## AERODYNAMIC POLE UPDATE

$$\beta_{\text{new}} = \beta_o + \alpha \delta \beta_i$$

## CONSTRAINTS ON AERODYNAMIC POLES

$$\beta \leq -0.01 \quad \text{AND} \quad \beta \geq \ln(0.005) / \Delta t$$

## PRELIMINARY RESULTS

Table I briefly describes some of the progress which has been made up to this time. Four sets of initially specified aerodynamic poles, associated coefficients and the initial cost are indicated as well as the minimum cost quantities. The first set of poles is a subset of the poles which were used by Dowell to generate the aerodynamic forces. Dowell's zero pole was not included for stability reasons and because the  $A_1$  term serves the same purpose of providing a constant term at  $t=0$ . The other sets of poles represent "random" selections between a small negative number and -1.0, -2.0 and -3.0.

A minimum cost was obtained for each of these sets of poles. The poles close to those of the generating function produced the lowest cost. Minimum cost increases from there as the range of initial poles widens. It is noted that finding a minimum isn't always guaranteed. For some sets of initial poles, the least-squares fit doesn't converge or the program determines a local maxima instead of a local minima.

One of the immediate observations which can be made from Table I is that the aerodynamic poles tend to decrease in magnitude as the cost is minimized. The same trend occurs as other rigid body forces are being approximated. The implication is that the fit improves at large times and degrades at small times. In terms of reduced frequencies, this means that the high frequency components of the curve are not being fit well. Thus, a weighted least-squares fit and a weighted square error function will be considered to improve the approximation at small times.

Finally, an assumption made in the quadratic interpolation subroutine which computes parameter step size is that when the square-error cost is computed for the "step-ahead" coefficients remain constant. From Table I, this appears to be a valid assumption, as over the entire range of parameter updates, coefficients have remained fairly unchanged.



# PRELIMINARY RESULTS

## LIFT DUE TO PLUNGE 6 POLE APPROXIMATIONS

Initial Data					Minimum Cost Results				
Poles*	Coefficients	A <sub>0</sub>	A <sub>1</sub>	Cost	Poles*	Coefficients	A <sub>0</sub>	A <sub>1</sub>	Cost
-0.1	-0.433	0.999	0.034	0.0170	-0.091	-1.24	0.999	-0.036	0.0012
-0.3	-0.51				-0.081	0.805			
-0.8	1.646				-0.148	-0.178			
-1.2	-2.841				-0.520	-0.116			
-1.75	2.19				-1.255	0.402			
-3.5	-0.574				-2.082	-0.123			
-0.2	-3.041	0.999	-0.074	0.0047	-0.185	-3.202	0.999	-0.056	0.0020
-0.4	43.06				-0.33	42.016			
-0.5	-124.97				-0.401	-114.617			
-0.6	121.388				-0.470	103.931			
-0.8	-51.012				-0.613	-37.816			
-1.0	14.162				-0.775	9.256			
-0.334	-6.036	0.999	-0.343	0.1995	-0.243	-5.681	0.999	-0.0997	0.0166
-0.668	46.844				-0.424	42.426			
-1.0	-178.225				-0.590	-152.419			
-1.334	333.826				-0.747	263.921			
-1.668	-299.231				-0.898	-215.887			
-2.0	102.678				-1.049	67.253			
-0.5	-9.818	0.999	-1.266	0.8607	-0.35	-6.406	0.999	-0.421	0.2469
-1.0	89.054				-0.683	42.793			
-1.5	-365.568				-1.03	-137.988			
-2.0	723.748				-1.409	222.239			
-2.5	-678.553				-1.837	-174.33			
-3.0	241.917				-2.343	53.628			

\* Poles indicated are  $\beta_i \frac{b}{V}$ .

Table I. Summary of some aerodynamic poles, coefficients and cost functions .

## PRELIMINARY RESULTS (CONTI)

Figure 2 illustrates some of the approximations to the rigid-body forces acting on a NACA0064 airfoil which are currently being obtained. The figure includes a pair of figures for each of four rigid-body aerodynamic forces. The lower figure in each pair contains a comparison between the aerodynamic data and an approximation made by using the initially specified aerodynamic poles. The norm square-error cost is indicated. The upper figure in each pair indicates the improved approximation after the minimization technique has been applied. Again, the minimum norm square-error cost is indicated. In all cases, aerodynamic data has been normalized with the largest absolute magnitude of force.

As can be seen, the technique does improve the approximation noticeably. In three of the four cases, the cost has been reduced by about 90%. In the case of "Moment Due to Pitch", the cost was observed to remain high even after minimization. This emphasizes the fact that the current method finds only the first extrema in cost. This extrema may be only a local extrema and not a global one.

## PRELIMINARY RESULTS

### NACA0064 RIGID BODY AERODYNAMIC FORCES

$$\beta_o = [-.334 \quad -.668 \quad -1.0 \quad -1.334 \quad -1.668 \quad -2.0]$$

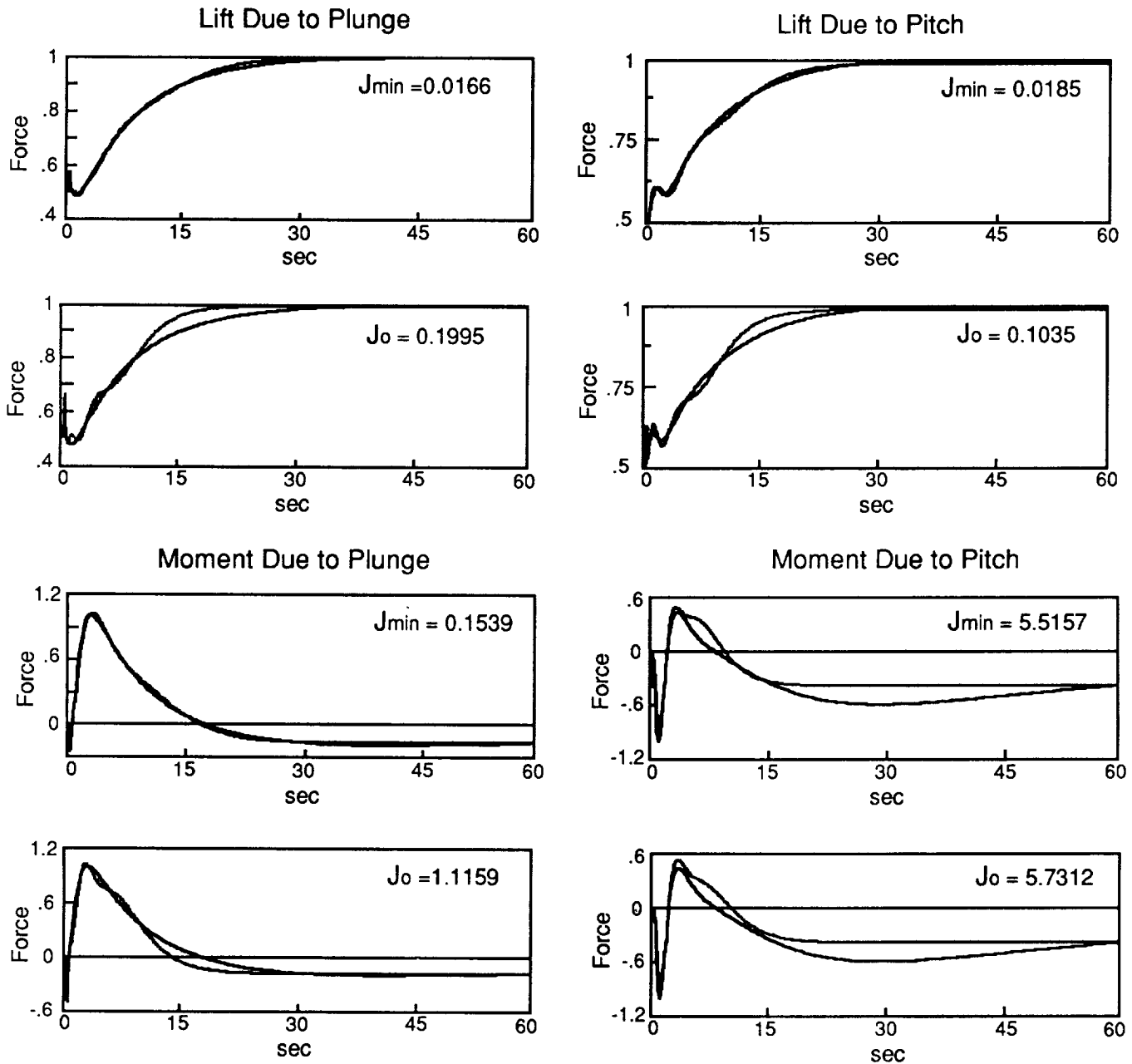


Figure 2. Approximations to NACA0064 airfoil rigid body forces using initial aerodynamic poles and aerodynamic poles computed for minimum cost.

## FURTHER DEVELOPMENT

Other methods will be considered to determine minimum cost. The method currently used is effective, but needs modification.

In an effort to improve the fit for small times, a weighted least squares fit will be implemented to determine the coefficients. A weighted square error cost function will also be considered.

Sometimes the program converges to a local maximum instead of minimum. Thus, means of forcing the program to converge on a minimum will be implemented.

## FURTHER DEVELOPMENT

- IMPROVE PROCEDURE TO IDENTIFY MINIMUM COST
- INVESTIGATE WEIGHTED LEAST SQUARES APPROXIMATION TO DETERMINE COEFFICIENTS
- INVESTIGATE A WEIGHTED SQUARE ERROR COST FUNCTION
- INVESTIGATE METHODS OF CHANGING THE SEARCH DIRECTION IF A MAXIMUM IS BEING APPROACHED INSTEAD OF A MINIMUM

## FURTHER APPLICATIONS

To further evaluate this technique, aerodynamic data generated for a real aircraft by a time-domain aerodynamic code in the subsonic and supersonic flight regimes will be modeled. Both rigid-body and flexible modes will be considered.

Finally, to fulfill the whole purpose of developing this model, methodology will need to be developed to integrate the aerodynamic model effectively with a structural model. Later, control systems will be integrated into the scheme. Using the integrated models, system stability will be evaluated.

## FURTHER APPLICATIONS

- APPLY TECHNIQUE TO FLEXIBLE AND RIGID BODY GENERALIZED AERODYNAMIC FORCES ACTING ON A REAL AIRCRAFT
- DEVELOP METHODOLOGY FOR INTEGRATING MODEL WITH DISCRETE-TIME STRUCTURAL MODEL AND PERFORMING STABILITY ANALYSIS FOR ARBITRARY MOTION

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